Finding the Shortest Synchronizing Sequence

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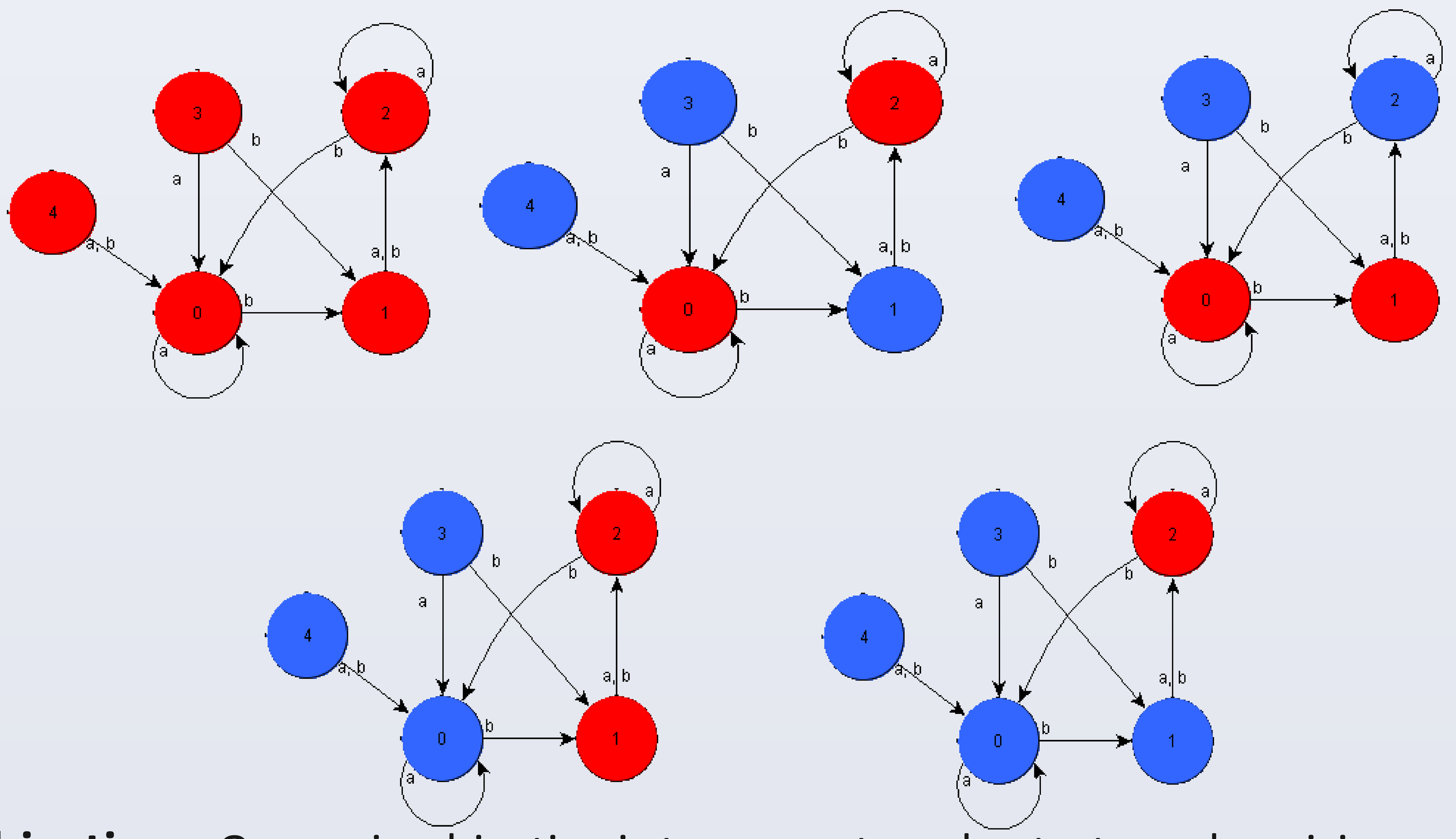
ABSTRACT

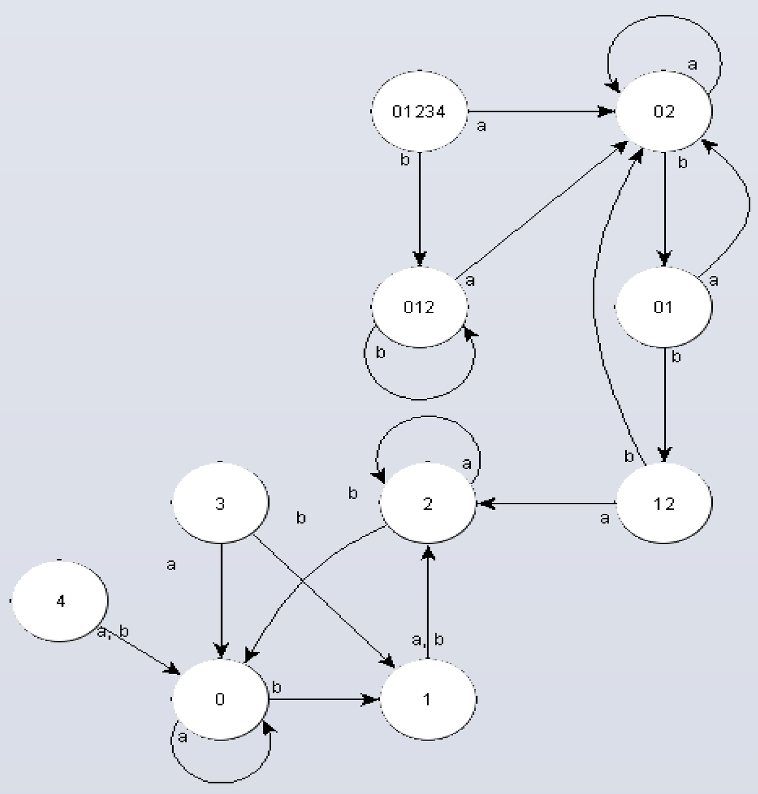
Cerný conjecture states that, for every automaton A with N > 1 states, the shortest reset word has the length $(N-1)^2$ (Cerný ,1964). We designed an algorithm to compute a shortest reset word for automata. The main idea is the use of a breadth-first-search and inverse-breadth-first-search to scan and compare the subsets. In our experiments, we used automata with up to N = 200 states and P = 2 input letters. We computed a shortest reset word for automata N = 200 states with P = 2 input letters under a minute on average.

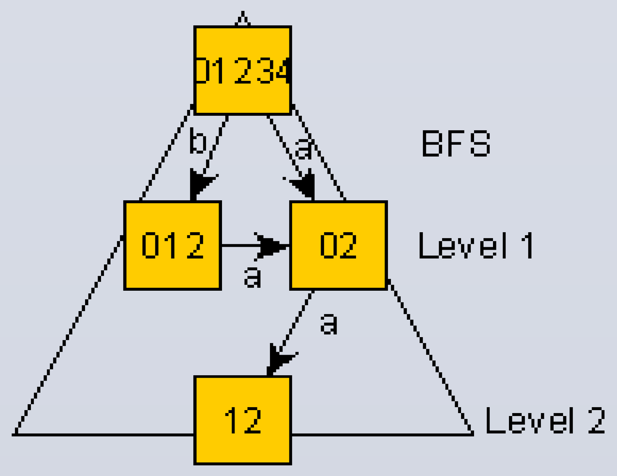


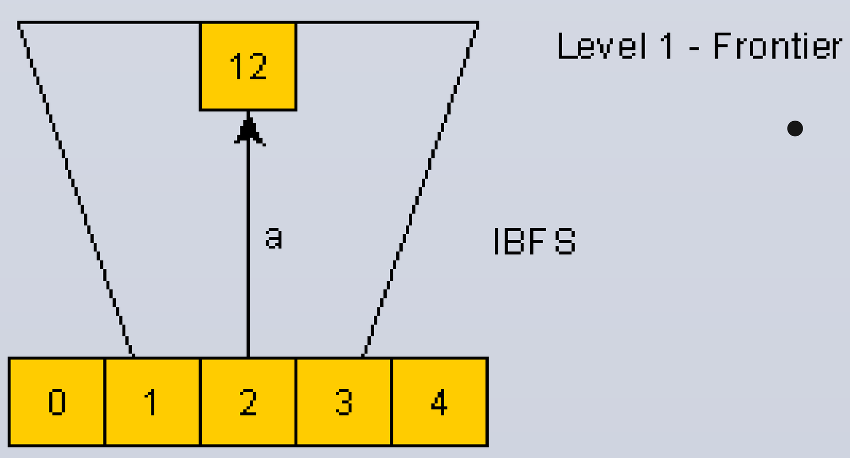
Objectives: Our main objective is to compute a shortest synchronizing sequence for automata with large number of states faster than currently existing algorithms.

THE ALGORITHM

- The algorithm we designed to compute a shortest reset word for synchronizing automata uses breadth-first-search(BFS) and inverse BFS. Queue structure is used for storing subsets.
- An automaton is synchronizing if and only if it admits a path from the universal set to a singleton set(Don and Zantema, 2017).







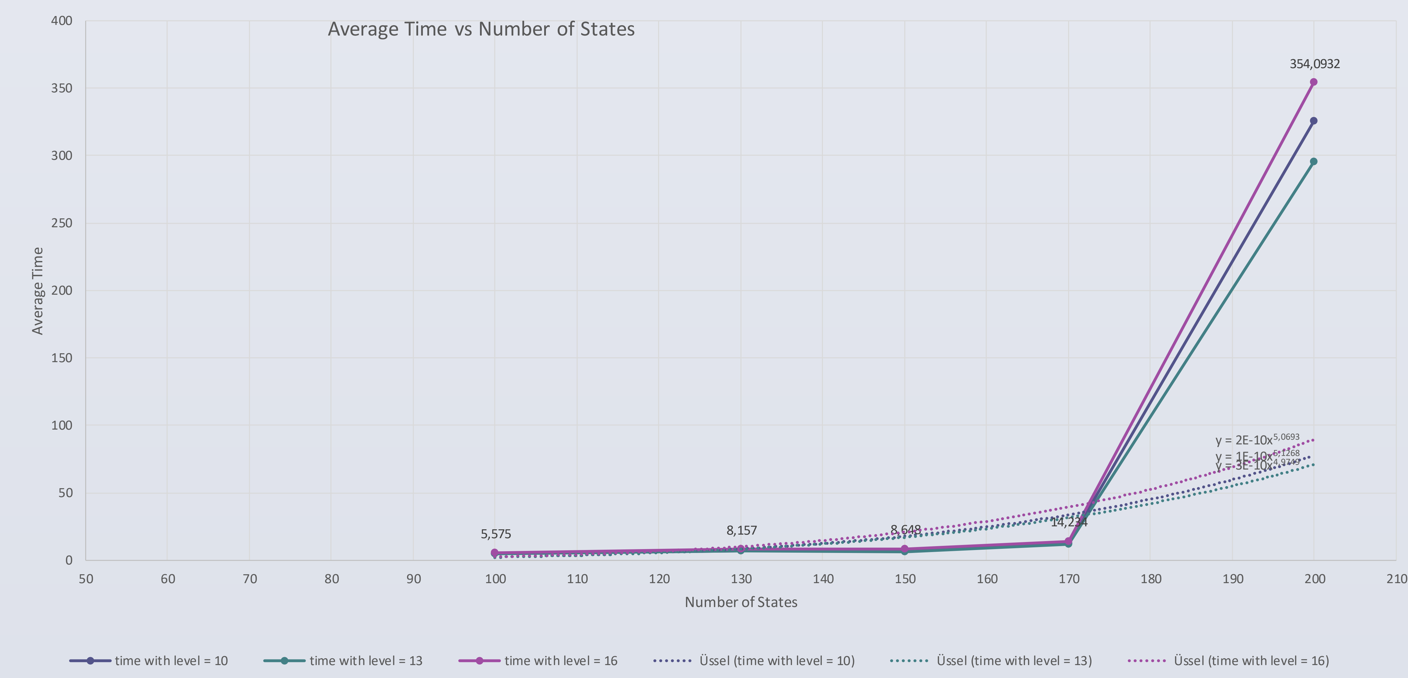
- First, the IBFS procedure starts from the sets with one member and advances upwards as many levels as wanted. Afterwards, BFS procedure starts on the power automaton, applying all the input letters to all the states in the automata and computing the successor images.
- For both BFS and IBFS, we hold all the subsets obtained within a queue structure since the start. Before adding a set S to a queue, we first check if S already exists in the queue.

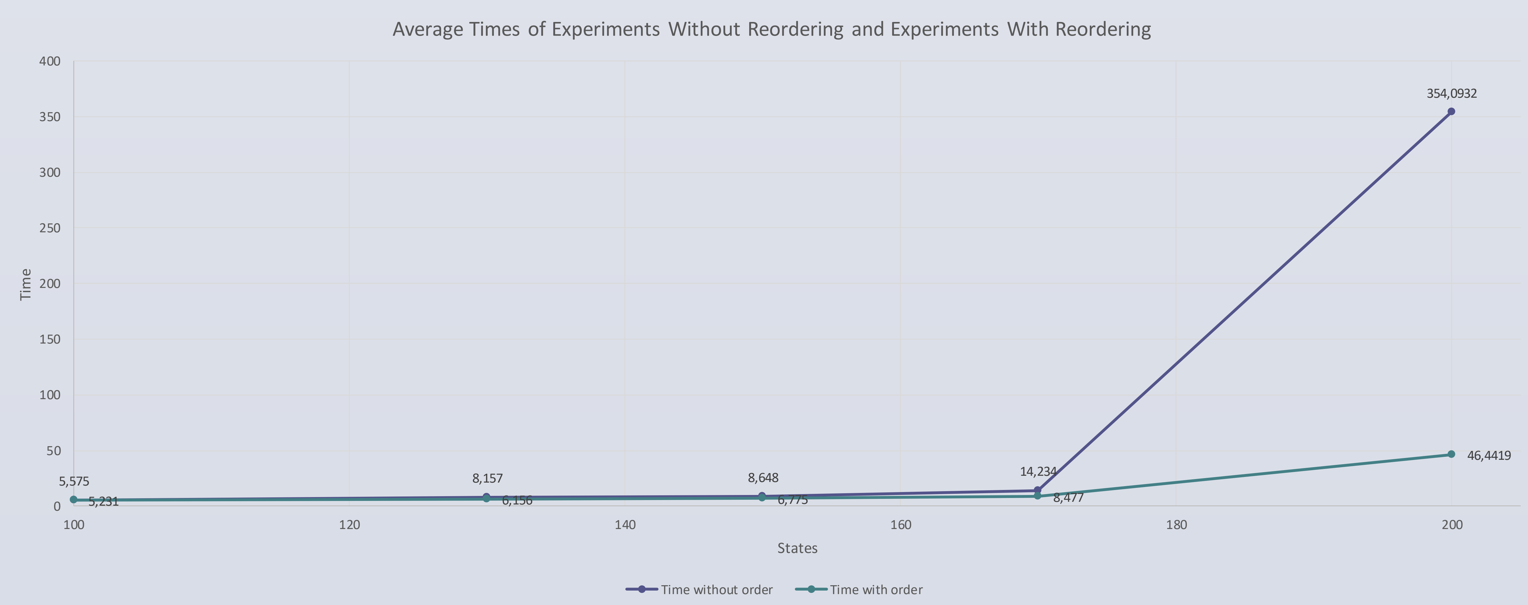
OPTIMIZATIONS

- Each subset is hashed before being stored. This makes the operation of reaching to subsets timewise less costly(Catalog).
- We have used two different catalogs to store our subsets, hash catalog and subset catalog. The hash catalog is used to check if a new subset exists in the queue. The subset catalog is used for subset and superset checks as subsets are inserted into the catalog according to their cardinality.
- Reordering of states according to frequency of being observed on paths was also implemented on our work.

RESULTS

- We perform our experiments on random automata with N = 100, 130, 150 ,170 and 200. For each N, 10 random automata with 2 input letters are generated.
- We tried all automata with level set to 10, 13 and 16. First graph shows tests without ordering.
- Second graph shows test with ordering, level set to 13, comparing with tests without ordering.





CONCLUSIONS

- Despite the improvements we have achieved on our work for 7 weeks, we could not reach our goal of computing the shortest word for automata with N ≥ 350 as the automata with N = 350 can be solved in 6 minutes on average (Kisielewicz, Kowalski, & Szykuła, 2013).
- We managed to design an algorithm to compute shortest reset word for automata with N = 200 and observe the effects of our algorithmic improvements on the time it takes to compute a shortest synchronizing sequence.

REFERENCES

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