MUB's (Mutually Unbiased Bases)

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ABSTRACT

Definition: Mutually Unbiased bases are orthonormal bases that the inverse of the dimension equals to value which is square of the magnitude of the inner product.

$$|\langle e_j | f_k \rangle|^2 = \frac{1}{d}, \quad \forall j, k \in \{1, \ldots, d\}.$$

Mutually Unbiased bases (MUBs) have been utilized in a few cryptographic and interchanges applications. There has been much hypothesis with respect to associations among MUBs and limited geometries. The greater part of which has concentrated on an association with projective and relative planes. We try to determine a way for deciding the maximum number of bases for arbitrary dimensions.

$$|e_j\rangle \text{ and } |f_k\rangle = \{|e_1\rangle, \ldots, |e_d\rangle\} \text{ and } \{|f_1\rangle, \ldots, |f_d\rangle\}$$

OBJECTIVES

- For two-dimensional space, three-dimensional space or four-dimensional space (d=2, d=3, d=4) there is no problem to find bases of MUB's however from six-dimensional space there is a problem to find all bases of MUB's, according to formula (d+1), there should be 7 bases whereas today only 3 of them have been found. With this project we try to find another bases for six- dimension, in addition to that to decide find the value that equal to maximum number of bases for arbitrary dimensions.

PROJECT DETAILS

Bases of MUB's for d = 2,

The three bases

$$M_0 = \{|0\rangle, |1\rangle\}$$
$$M_1 = \left\{\frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}}\right\}$$
$$M_2 = \left\{\frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \frac{|0\rangle - i|1\rangle}{\sqrt{2}}\right\}$$

Bases of MUB's for d = 4,

$$M_0 = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$$
$$M_1 = \left\{\frac{1}{2}(1,1,1,1), \frac{1}{2}(1,1,-1,-1), \frac{1}{2}(1,-1,-1,1), \frac{1}{2}(1,-1,1,-1)\right\}$$
$$M_2 = \left\{\frac{1}{2}(1,-1,-i,-i), \frac{1}{2}(1,-1,i,i), \frac{1}{2}(1,1,i,-i), \frac{1}{2}(1,1,-i,i)\right\}$$
$$M_3 = \left\{\frac{1}{2}(1,-i,-i,-1), \frac{1}{2}(1,-i,i,1), \frac{1}{2}(1,i,i,-1), \frac{1}{2}(1,i,-i,1)\right\}$$
$$M_4 = \left\{\frac{1}{2}(1,-i,-1,-i), \frac{1}{2}(1,-i,1,i), \frac{1}{2}(1,i,-1,i), \frac{1}{2}(1,i,1,-i)\right\}$$

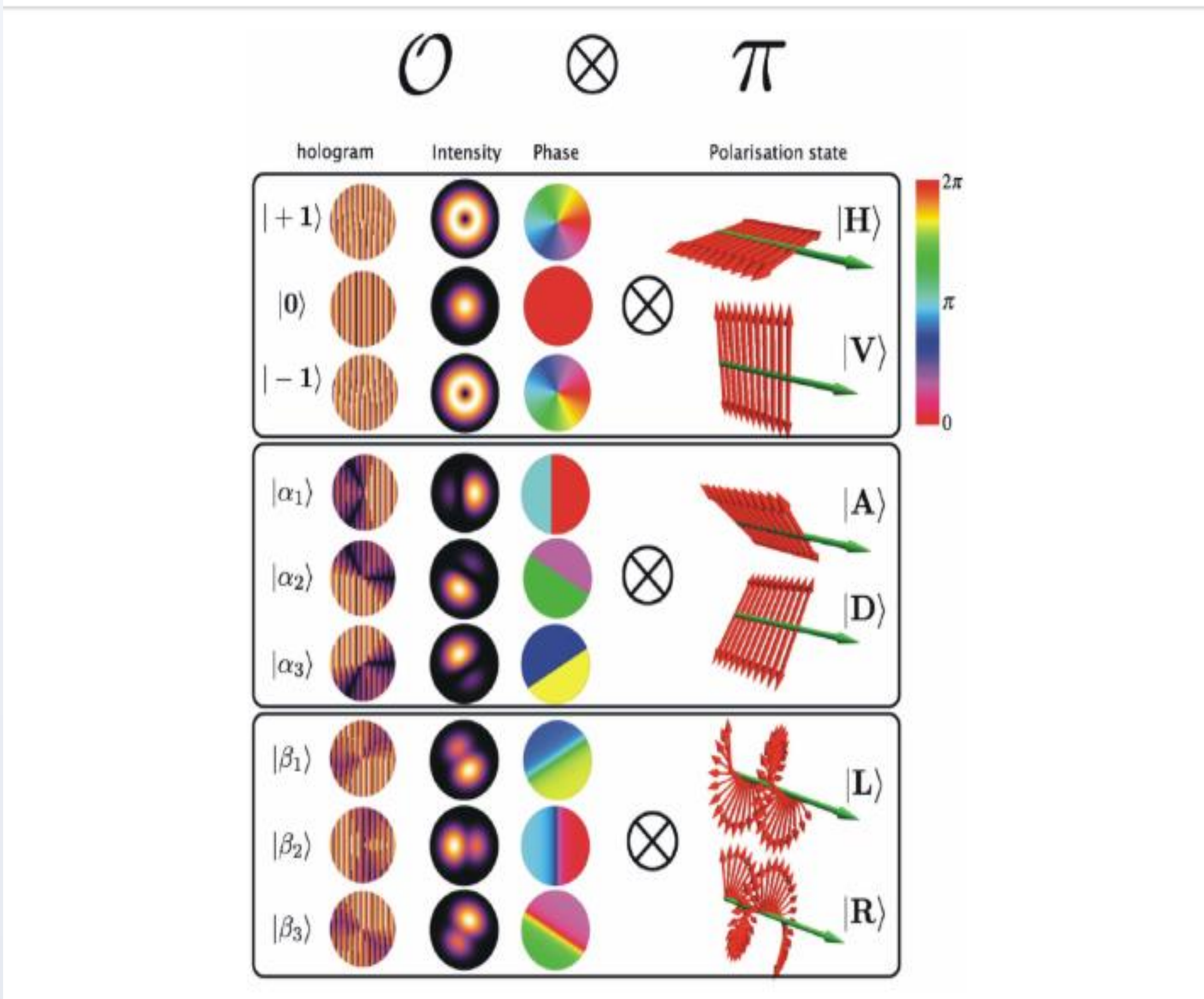
Mutually Unbiased Bases (MUBs) are vital in quantum data hypothesis. While developments of complete arrangements of d + 1 MUBs in Copfd are known when d is a prime power, it is obscure if such total sets exist in non-prime power measurements. It has been guessed that arrangements of complete MUBs possibly exist in Copfd if a projective plane of size d additionally exists. We research the structure of MUBs utilizing two logarithmic devices: connection algebras and gathering rings. We develop two connection algebras from MUBs and contrast these with connection algebras built from projective planes. We demonstrate a few instances of complete arrangements of MUBs in Copfd, that when considered as components of a gathering ring structure a commutative monoid. We guess that total arrangements of MUBs will dependably shape a monoid if the suitable gathering ring is picked.

$\circ$	I_x	O_x	N_{xy}	N_{yx}	N_{yz}
I_x	I_x	O_x	N_{xy}	N_{yx}	N_{yz}
O_x	O_x	$O_x + I_x$	N_{xy}	$\emptyset$	$\emptyset$
N_{xy}	$\emptyset$	$\emptyset$	$\emptyset$	$O_x + I_x$	N_{xz}
N_{yx}	N_{yx}	N_{yx}	$O_y + I_y$	$\emptyset$	$\emptyset$
N_{yz}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

$\hat{*}$	0	$x1$	$x2$	$y1$	$y2$	$z1$	$z2$
0	0	0	0	0	0	0	0
$x1$	0	$x1$	$x2$	$y1$	$y2$	$z1$	$z2$
$x2$	0	$x2$	$x1$	$y1$	$y2$	$z2$	$z1$
$y1$	0	$y1$	$y1$	$y1$	0	$y1$	$y1$
$y2$	0	$y2$	$y2$	0	$y2$	$y2$	$y2$
$z1$	0	$z1$	$z2$	$y1$	$y2$	$x2$	$x1$
$z2$	0	$z2$	$z1$	$y1$	$y2$	$x1$	$x2$

PROJECT DETAILS II

Bases of MUB's that have been for d = 6,



MUBs for hybrid photonic qusix encoding: Representation of quantum states with dimension d = 6 obtained from the direct product of a three-dimensional subspace $\mathcal{O}$ of OAM and the two-dimensional space π of polarization. The three main boxes correspond to the three MUBs. On the left side, the intensity and phase distributions of each OAM spatial mode and the corresponding generating kinoform are shown. On the right side the polarization states are illustrated graphically by showing the optical electric field orientation in space at a given time.

CONCLUSION

Commonly fair bases are essential natives in quantum data hypothesis. They have applications in quantum cryptography and the plan of ideal estimations. It is realized that in measurement d at most d+ 1 commonly impartial bases can exist.

$$\left|v_k^{(0)}\right\rangle_j = \delta_{jk},$$

$$\left|v_k^{(1)}\right\rangle_j = \frac{1}{\sqrt{N}} e^{\frac{2\pi i}{N}(j^2+jk)},$$

$$\vdots$$

$$\left|v_k^{(r)}\right\rangle_j = \frac{1}{\sqrt{N}} e^{\frac{2\pi i}{N}(rj^2+jk)},$$

$$\vdots$$

$$\left|v_k^{(N-1)}\right\rangle_j = \frac{1}{\sqrt{N}} e^{\frac{2\pi i}{N}((N-1)j^2+jk)},$$

$$\left|v_k^{(N)}\right\rangle_j = \frac{1}{\sqrt{N}} e^{\frac{2\pi i}{N}jk}.$$

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