KANADE-RUSSELL'S IDENTITYFINDER

STUDENTS / UNIVERSITIES

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INTRODUCTION

Integer partitions: Is a way of writing any positive integer as a sum of positive integers.

5: [5] , [4,1] , [3,2] , [3,1,1] , [2,2,1] , [2,1,1,1] , [1,1,1,1,1]

Identities: If we restrict partitions, identities may occur.

Euler Pair

$$p(n\,|\,odd\ parts) = p(n\,|\,distinct\ parts)$$

For n=4;

Partitions with odd parts: [3,1] , [1,1,1,1] count:2

Partitions with distinct parts: [3,1] , [4] count:2

$1+1$	2
$1+1+1$ 3	3 $2+1$
$1+1+1+1$ $3+1$	4 $3+1$

?=?

Euler's Identity

$$p(n\,|\,odd\ parts) = p(n\,|\,distinct\ parts)$$

$$\prod_{n=1}^{\infty}\frac{1}{(1-q^{2n-1})} = \prod_{n=1}^{\infty}(1+q^{n})$$

Roger-Ramanujan's First Identity

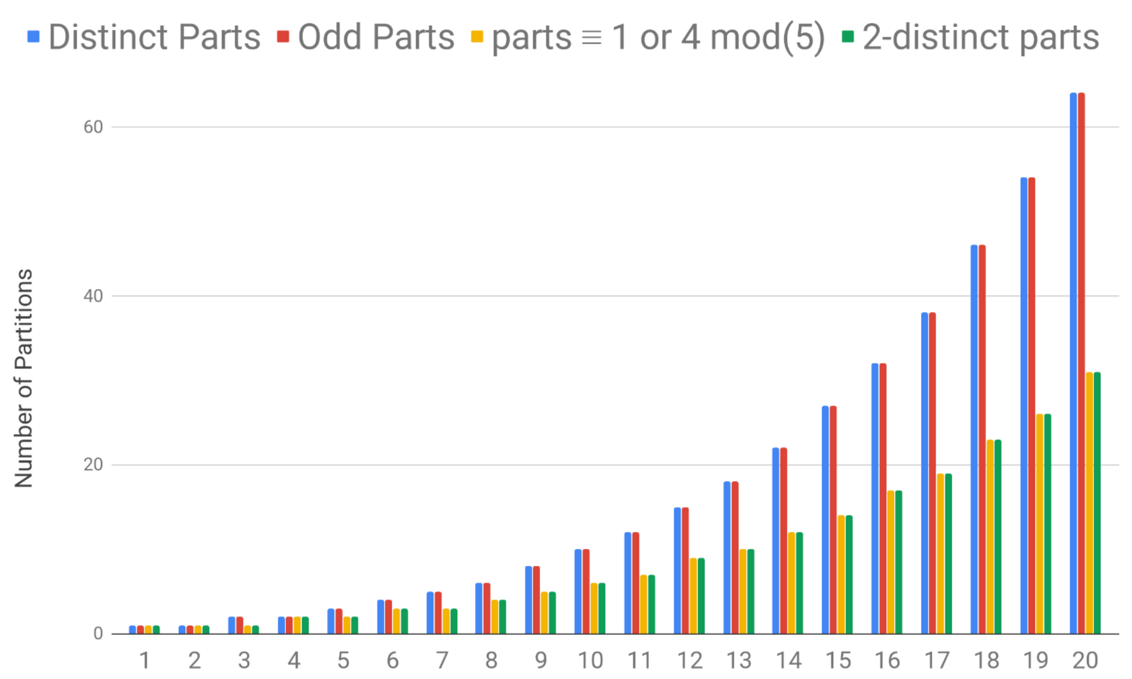
p(n I parts= 1 or 4 (mod 5)) = p(n I 2-distinct parts)

For n=6;

Partitions with congruent to 1 or 4 modulo 5: [1,1,1,1,1,1] , [4,1,1] , [6]

Partitions with 2-distinct parts: [6] , [5,1] , [4,2]

$$\prod_{n=1}^{\infty}\frac{1}{(1-q^{5n-4})(1-q^{5n-1})}$$



Kanade-Russell's First Identity

p(n | The number of partitions of a non-negative integer into parts congruent to ±1 or ±3 (mod 9))

=

p(n | Partitions with difference at least 3 at distance 2 such that if two consecutive parts differ by at most 1, then their sum is divisible by 3)

For n = 10 : [5,4,1], [6,3,1], [6,4], [7,2,1] ,[7,3], [8,2], [9,1], [10]

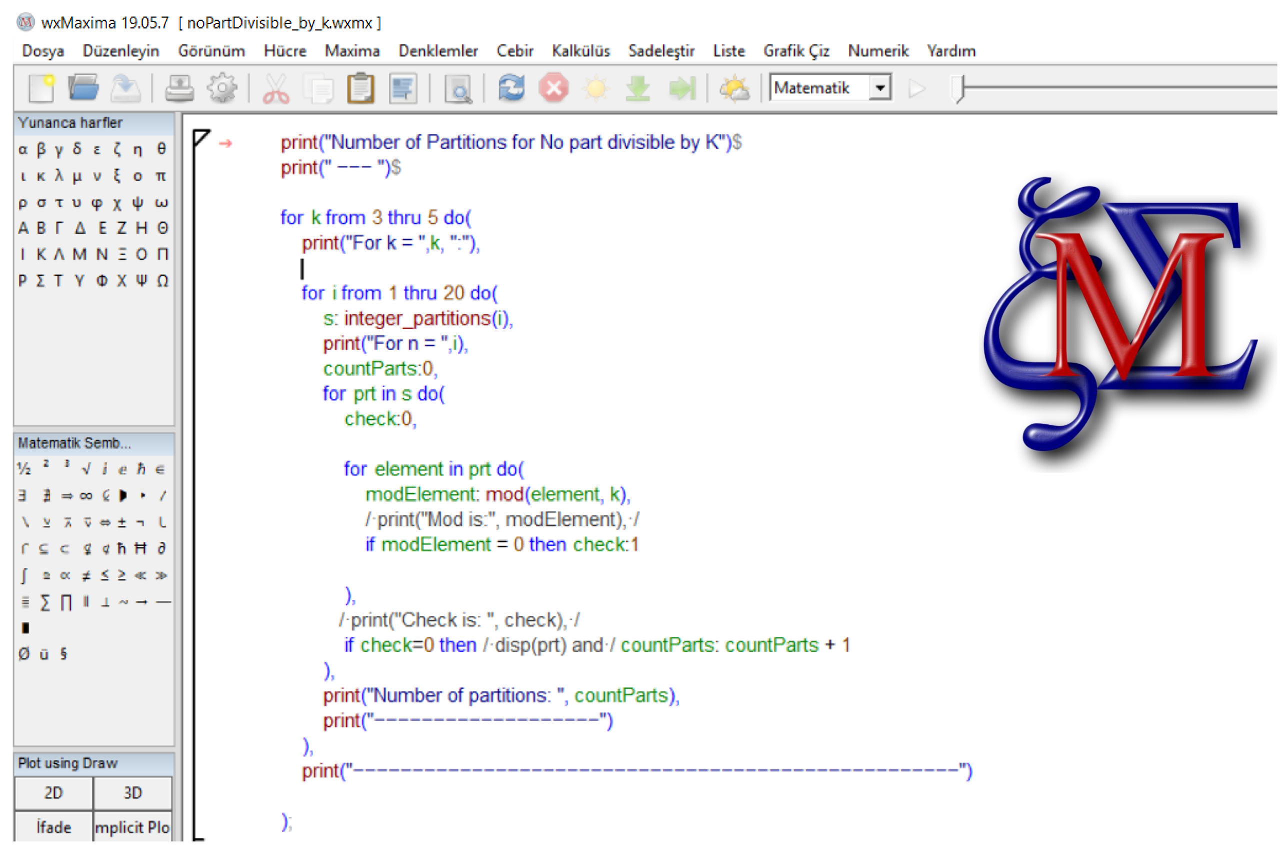
AIM

To contribute to the subject by discovering a new identity.

And learning the subject through research.

METHOD

MAXIMA



Step 1

- Get familiar with integer partitions by studying Integer Partitions by George E. Andrews and Kimmo Eriksson

Step 2

- Found number of partitions of identities mentioned in the book in first 5 chapters from N=1 to N=20 through coding in Maxima to get familiar with functional programming.

Step 3

- Study the paper "IdentityFinder and Some New Identities of Rogers–Ramanujan Type" by S. Kanade and M. C. Russell
- Implement algorithm explained by the paper through Maxima

Step 4

By analyzing more identity theorems, tried to discover or rediscover identities. Still do...

CONCLUSION

- Learning integer partitions
- Learning how to use maxima and coding identities mentioned in the book
- Reading paper by Kanade-Russell and code the algorithm
- Try to find our own identities

REFERENCES

- -Andrews, George E., and Kimmo Eriksson. Integer Partitions. Cambridge: Cambridge University Press, 2004.
- -Andrews, George E. 1984. The Theory of Partitions. Section, Number Theory. Cambridge [Cambridgeshire]: Cambridge University Press. http://search.ebscohost.com/login.aspx?direct=true&db=e000xww&AN=570384&site=ehost-live.
- -Andrews, George E. "An Analytic Proof of the Rogers-Ramanujan-Gordon Identities." American Journal of Mathematics88, no. 4 (1966): 844-46. doi:10.2307/2373082.
- -Kanade, Shashank, and Matthew C. Russell. "IdentityFinde rand Some New Identities of Rogers–Ramanujan Type." Experimental Mathematics24, no. 4 (2015): 419-23. doi:10.1080/10586458.2015.1015186.
- -Alladi, Krishnaswami. "A Combinatorial Correspondence Related to Göllnitz' (big) Partition Theorem and Applications." Transactions of the American Mathematical Society 349 (July 1997): 2721-735. doi:https://doi.org/10.1090/S0002-9947-97-01944-2.