Error Correcting Codes from Finite Geometry

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ABSTRACT

Error correcting codes are used for detecting and correcting errors in the transmission of data in many areas such as IBAN, ISBN etc. One of the ways of obtaining error correcting codes is using geometric objects such as projective planes and generalised quadrangle. We examined the properties of those objects and obtained all the codewords. While doing that we studied their mathematical structures and checked our result via SageMath. Towards the end of the project we looked at some applications of error correcting codes such as ISBN, TR citizenship number, IBAN and zipping file.

OBJECTIVES

In this project, our main purpose is to obtain error correction and detection codes from various geometric objects such as the Fano plane and generalised quadrangle and to implement those codes via SageMath.

IMPLEMENTATIONS

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A codebook is described below
((0, 0, 0, 0, 0, 0, 0), 'Yes')
((1, 0, 0, 0, 0, 1, 1), 'No')
((0, 1, 0, 0, 1, 0, 1), 'Leave')
((1, 1, 0, 0, 1, 1, 0), 'Help')
((0, 0, 1, 0, 1, 1, 0), 'Repeat')
((1, 0, 1, 0, 1, 0, 1), 'Roger')
((0, 1, 1, 0, 0, 1, 1), 'Danger')
((1, 1, 1, 0, 0, 0, 0), 'Enemy')
((0, 0, 0, 1, 1, 1, 1), 'Ally')
((1, 0, 0, 1, 1, 0, 0), 'Safe')
((0, 1, 0, 1, 0, 1, 0), 'Alpha')
((1, 1, 0, 1, 0, 0, 1), 'Beta')
((0, 0, 1, 1, 0, 0, 1), 'Defend')
((1, 0, 1, 1, 0, 1, 0), 'Attack')
((0, 1, 1, 1, 1, 0, 0), 'Victory')
((1, 1, 1, 1, 1, 1, 1), 'Defeat')
To sending a message such as 'Yes', the codeword (0, 0, 0, 0, 0, 0, 0)
must be sent which need 7 bits memory or we can use a feature of
codewords..
But we can use less memory space. For example, if the message 'Roger'
should be sent (1, 0, 1, 0, 1, 0, 1)
Compressed version of the code is..
(1, 1, 0, 1)
So sending 'Roger' message 4 bits would be enough and it can be
amplified at the target area of message
Amplifying the message...
(1, 0, 1, 0, 1, 0, 1)
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One of the applicaitons of error correcting codes – Zipping File

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The main message which is intended to sent:
(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0)
The incorrect message which one error is implied on it
(0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0)
The Decoded/Corrected Code
(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0)
Weight 2 error implied code:
(0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0)
Corrected Code:
(1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1)
The main message:
(0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0)
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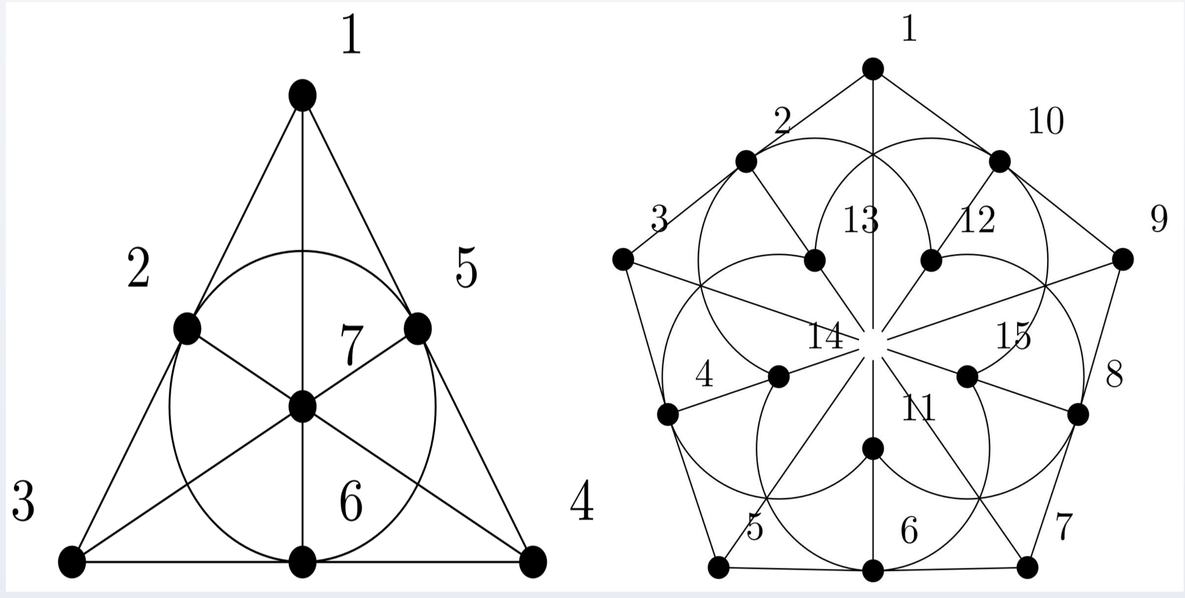
One of the applicaitons of error correcting codes - Decoding

FANO PLANE AND GENERALISED QUADRANGLE



Error detection is a process in which the error and distortion of data due to channel noise is detected. Error correction is the process of finding and correcting errors in transmitted data. Various ways to obtain error correcting code have been invented. In particular, error correcting codes can be obtained by using projective planes.

As for the Fano plane, by examining the distances between codewords, we found that the minimum distance between two codewords is 3. Therefore, if there is one error, it can be detected. It can be corrected by choosing the closest codeword to the error vector. In case of two errors, the error can still be detected, but cannot be corrected. This is because there are two possible codewords with the same distance to the error vector and we cannot know which one of these is the correct codeword.

CONCLUSION AND FURTHER WORK

Step	Modulo 11 check digit calculation									
1. Weight each digit	**Weighting**	10	9	8	7	6	5	4	3	2
2. Multipy digit by wieighting	**ISBN**	0	3	4	0	8	6	9	1	9
	Result	0	27	32	0	48	30	36	3	18
3. Add results	Sum	0 + 27+ 32 + 0 + 48 + 30 + 36 + 3 + 18 = 194								
4. Divide the sum by 11 keep the remlander	194 divide 11 = 17 remainder 7									
5. Take remainder from 11	11 minus 7 = 4									

(www.ict4u.net)

We learned how to implement codewords from geometric shapes such as the Fano plane and the generalised quadrangle into many areas such as transmission of messages, zipping files or IBAN and ISBN. When we consider geometric objects with more points or lines, the number of codewords increases. For example, going from the Fano plane to the generalised quadrangle, the number of codewords increased from 16 to 1024. More complex structures can be used when more codewords are needed for bigger data.

REFERENCES

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