# SKETCHES ON SINGLE BOARD COMPUTERS 

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#### Abstract

Data sketches are probabilistic data structures with mathematically proven error bounds that store information about the datasets using small amount of space using hash functions. They can approximate some predefined questions (i.e. cardinality or frequency estimation) about big data with high accuracy. We chose a data sketch called HyperLogLog and tested it with many different configurations in order to find results that show least errors possible when estimating unique elements in large genomic datasets.


Abstract

Keywords: Big Data, Data Sketches, Bioinformatics, HyperLogLog, K-Mers

## 1 Introduction

Estimating the number of unique items in a dataset is called the count-distinct problem. A naïve approach to this would be to keep track of each unique member encountered. However, it becomes a challenge for memory and speed when a big data is processed; it isn't always practical to calculate the exact cardinality. Data sketches or probabilistic data structures are employed to overcome the inefficient use of resources and approximate cardinalities or frequencies with mathematically proven error bounds in a large datasets. These methods are sufficiently accurate when some amount of error is acceptable. The lecture notes of Chakrabarti (2015) presented the basics of the data sketches in a very concise fashion. Some of the most prominent data sketches in the field are HyperLogLog (for cardinality estimation) and Count-Min (for frequency estimation). In this work, we have focused on the count-distinct problem, and therefore HyperLogLog algorithm. This algorithm was proposed by Flajolet et al. (2007) as an extention to LogLog algorithm (Durand and Flajolet, 2003) which was derived from the Flajolet-Martin algorithm (Flajolet and Martin, 1985). HyperLogLog is well-known for its low memory footprint and high accuracy on large number of cardinalities. Heule et al. (2013) presented some improvements for HyperLogLog in order to reduce memory requirements and increase accuracy for some cases of cardinalities.

In the field of Bioinformatics, large genomic datasets are stored through DNA sequencing to be processed later on. Such datasets could reach GBs or even TBs in size. In order to perform
bioinformatics analysis, substrings or subsequences of length k called " k -mers" are read from DNA sequences. It is important to determine the cardinalities or frequencies of certain k-mers in such datasets, however, the computational resources might be limited or insufficient, such as in the single board computers. In this case, data sketches can help immensely to overcome these constraints by estimating with the cost of accuracy.

Zhang et al. (2014) presented the khmer software package for frequency estimation of k-mers using Count-Min sketch and compared the performances of several k -mer counting packages with their proposed model. Rizk et al. (2013) proposed an exact k-mer counting software called DSK (disk streaming of $k$-mers) which requires a user-defined amount of memory and disk space. We deployed DSK in our work and compared our approximated results with the exact results obtained from DSK. Mohamadi and Birol (2017) proposed ntCard algorithm for estimating k-mer frequencies using ntHash (Mohamadi et al., 2016). During our research, we deployed and analyzed the ntCard algorithm to make speed and accuracy comparisons with the results obtained from our model.

Just like in many other data sketches, a hash function is used in the HyperLogLog algorithm. Since there are many different hash functions used in different conditions, we had to narrow down our options to few non-cryptographic and considerably fast hash functions: Murmur3 Hash, Tabulation Hash, City Hash and Spooky Hash. Instead of picking just one, we decided to compare the results of HyperLogLog algorithm working with these hash functions.

The genomic datasets that we used to conduct our experiments with HyperLogLog were obtained from UCSC Genome Database (Karolchik et al., 2003). We used the four datasets: danRer11, triCas2, apICal1 and droEre2. aplCal (A. californica) 1 has size 697Mb, Danrer (D. rerio) as biggest data of our sample has size of 1.6 Gb . DroEre2 (D. erecta) is 149 Mb , triCas2 (T. casteneum) is 195 Mb . Additionally, we have tested our model with 2 more datasets called est and est 1 . Data sizes of est and est 1 are 97 Mb and 89 Mb respectively.

For the sake of consistency, we conducted our experiments on a single HPC server with the following specifications: $2 \times \operatorname{Intel}(\mathrm{R}) \mathrm{Xeon(R)}$ CPU E5-2620 v4 @ 2.10 GHz 16 cores running Ubuntu 16.04.2 LTS.

In the rest of the report, we mention the HyperLogLog algorithm in detail, give the percent error results for each hash function with the corresponding K and bucket bit values, present our outcome from the results, and finally give our conclusion and present potential future work.

## 2 HyperLogLog on Genomic Data



Figure 1: ("How to count distinct?" 2015)

Our implementation of HyperLogLog offers different combinations of hash functions, register sizes (or bucket bits) and k-mers. Program asks the user to choose a hash function between MurMur, Tabulation, Spooky and City Hash, then expects number of bits to construct buckets and $k$ number to estimate cardinality of a particular k-mer size.

HyperLogLog operates as follows: after getting the number of bits (call it p ) from user input, construct an array of size $2^{\mathrm{p}}$. Later, the dataset whose cardinality will be estimated is read to be hashed. 32 or 64 -bit value of a specific element that returned from hash function is used to determine bucket index and probabilistic calculation of cardinality. First binary value m bits of returned hash value are used to determine bucket index (see Figure 1). Number of leading zeros of remained bits will be the value of that bucket index if it is the maximum value that have been seen so far. Every bucket represents a subset of the initial datasets. Values of that buckets gives us a probabilistic intuition to calculate estimation. Each leading zero encountered gives us to a reliable conviction about doubling of number of distinct elements of that subset. Evaluating value of all buckets together gives an accurate estimation.

$$
E:=\alpha_{m} \cdot m^{2} \cdot\left(\sum_{j=1}^{m} 2^{-M[j]}\right)
$$

## where

$$
\alpha_{m}:=\left(m \int_{0}^{\infty}\left(\log _{2}\left(\frac{2+u}{1+u}\right)\right)^{m} d u\right)^{-1}
$$

Figure 2: ('HyperLogLog in Practice: Algorithmic Engineering of a State-of-the-Art Cardinality Estimation Algorithm," 2017)

The formula seen above in Figure 2 gives us the final cardinality estimate(E) is produced from the substream estimates as the normalized bias corrected harmonic mean where $m$ is equal to number of buckets( $2^{\mathrm{p})} \mathrm{M}$ is the array of buckets and $\alpha_{\mathrm{m}}$ is the constant (also known as magic number).
In bioinformatics, researchers need to work with different K-Mer sizes, and as K-Mer size increases calculations become difficult, because of this we started with length 32 and worked our way up to 64. We had 6 different data, some were larger around 1 GB and some were lower around 70 mb . We tried to find a correlation between the hash functions and accuracy of the sketch. For this we worked with different parameters, such as length of our data by using different data sizes. Working with different bucket sizes, working with different K-Mers and finally changing the hashes themselves.
Since the files were too large for us to find the cardinality on our own, we used a specific tool that does that. Namely DSK. It is an exact counting tool that's uses the disk to count and find the cardinality. The question becomes then why anyone would use data sketches and not DSK for cardinality counting, and the answer is because DSK can take a long time and requires a lot of space.

We run our experiments by using 32 -bit hash functions on $\mathrm{k}=32,40,48$, 56 and 64 , but 64 -bit versions of them also available. Following sections present result of experiments on 32 and 64 k mer. Other results and corresponding plots can be found in Appendices section.

### 2.1 Percent Error Results with K-Mer Length 32

By looking at the tables with K-Mer length 32, we can see a lot of fluctuations in the accuracies of hash functions.

| ERR | Hash 1 (City) |  |  |  |  | AVG | STDEV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% | 6 - | - | 10 | 12 | 14 |  |  |
| aplCal1 | 28,32903 | -0,47752 | 0,32568 | -2,42212 | -2,69611 | 4,360159 | 13,31973 |
| droEre2 | 26,08472 | 9,007605 | 1,537999 | -1,8448 | -1,86672 | 32,91881 | 10,52685 |
| triCas2 | -11,3154 | -6,98836 | 3,641623 | 1,016213 | 1,644769 | $-2,40023$ | 6,423955 |
| est | -11,1559 | 16,44484 | 23,94884 | 22,94607 | 22,82548 | 15,00186 | 14,92213 |
| est1 | 14,80821 | 9,404392 | 13,07921 | 15,97663 | 15,39878 | 13,73344 | 2,651841 |
| danRer | -23,5561 | -4,5972 | 4,481148 | 6,296602 | 5,481442. | -2,37883 | 12,62444 |

Table 1:32 K-Mer Length City Hash Results

|  | Hash 2 (Spooky) |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\%$ | 6 | 8 | - | 10 | 12 | 14 | AVG |
| STDEV |  |  |  |  |  |  |  |
| aplCal1 | $-6,16254$ | $-2,00645$ | $-3,62256$ | $-5,16196$ | $-3,25258$ | $-4,04122$ | 1,635051 |
| droEre2 | 4,120409 | 3,588149 | $-2,51519$ | $-4,48429$ | $-1,55365$ | $-0,16891$ | 3,826184 |
| triCas2 | $-11,8826$ | $-10,2646$ | $-2,74888$ | $-1,3645$ | 1,601884 | $-4,93174$ | 5,8509 |
| est | $-10,4284$ | 0,804696 | 12,94201 | 18,33887 | 22,61821 | 8,855077 | 13,5345 |
| est1 | 20,01498 | 26,09756 | 16,50607 | 19,32839 | 16,80599 | 19,7506 | 3,864255 |
| danRer | $-3,41148$ | 13,75741 | 14,05929 | 6,6609 | 7,035217 | 7,620267 | 7,107877 |

Table 2: 32 K-Mer Length Spooky Hash Results

|  | Hash 4 (Murmur) |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\%$ | 6 | 8 | 10 | 12 | 14 | AVG | STDEV |
| apICal1 | 18,15211 | 9,009887 | 1,382242 | $-3,04857$ | $-2,22886$ | 4,653363 | 8,924408 |
| droEre2 | 10,6928 | 3,861166 | $-3,92286$ | $-1,46581$ | $-2,67153$ | 1,298752 | 6,030865 |
| triCas2 | 5,122158 | 11,19662 | 4,075823 | 0,880328 | 1,417772 | 4,53854 | 4,1233 |
| est | $-11,9336$ | 9,145206 | 22,65226 | 23,08353 | 22,12962 | 13,0154 | 15,12232 |
| est1 | 2,439208 | 11,3283 | 15,98586 | 16,92235 | 15,31637 | 12,39842 | 5,961988 |
| danRer | $-2,84937$ | 4,091096 | 11,73827 | 8,783472 | 7,06333 | 5,765358 | 5,556216 |

Table 3: 32 K-Mer Length Murmur Hash Results

|  | Hash 5 (Tabulation) |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\%$ | 6 | 8 | 10 | 12 | 14 | AVG | STDEV |
| aplCal1 | $-0,81926$ | $-0,31408$ | $-0,95929$ | $-3,14298$ | $-1,57649$ | $-1,36242$ | 1,092333 |
| droEre2 | $-2,40244$ | 0,668195 | $-6,52774$ | $-1,18951$ | $-1,8116$ | $-2,25262$ | 2,653063 |
| triCas2 | $-19,5032$ | 3,875637 | 0,598706 | 4,286389 | 2,253572 | $-1,69777$ | 10,05973 |
| est | 15,23084 | 7,434267 | 17,38598 | 23,71222 | 22,20439 | 17,19354 | 6,456285 |
| est1 | 18,61013 | 13,28527 | 12,86896 | 15,49735 | 13,87788 | 14,82792 | 2,338436 |
| danRer | $-10,8732$ | 1,804279 | 4,676993 | 7,592755 | 7,263135 | 2,092788 | 7,613489 |

Table 4: 32 K-Mer Length Tabulation Hash Results

We can see all hash functions perform better for larger amounts of bits taken. This means that we can reach a more accurate result by setting up an array of $2^{\wedge} 10$ or $2^{\wedge} 14$ buckets to reach our optimal accuracy with bits taken. Yet in the other parameters things are not as clear. Looking at the 10 bits column of tables 1, 2, 3, 4 we can see City hash performs the best for "aplCal 1 " and Tabulation Hash for triCas2, they also seem to perform the best overall. Yet while all the hashes in this 10 bits column perform around below $10 \%$ we have 2 datasets (est and est1) that has almost $20 \%$ error rate. This at first led us to think our algorithm was not very accurate because $20 \%$ error is too high for our expectations. This led us to test all these data with another, very sophisticated cardinality counting algorithm for K-Mer counting called NTCard, and the results were better than HyperLogLog, yet similar.

| ERROR - | 32 | 40 | 48 | 56 | 64 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| triCas2 | 0.351716 | 0.38153 | 0.48135 | 0.5399386 | 0.704322 |
| est | 14.264171 | 15.52537 | 14.83173 | 16.830817 | 17.24306 |
| est1 | 7.0655798 | 7.579474 | 8.014691 | 8.6266012 | 8.858206 |
| danRer11 | 0.140161 | 0.177882 |  |  |  |
| droEre2 | 0.2209904 | -0.02371 | 0.104777 | 0.1375697 | 0.261346 |
| aplCal1 | -0.004707 | 0.105699 | 0.098857 | 0.0807122 | 0.09226 |

Table 5: NTCard Results for K-Mer Lengths 32, 40, 48, 56 and 64
By looking at Table 5, we can see even though ntCard performed really well for triCas2 and Danrer11, it failed to perform accurately when it came to "est" and "est1". This proved that the problem was not with our HyperLogLog implementation. Unfortunately, we were not able to obtain DSK results with the danRer11 dataset with K values of 48,56 and 64 , therefore the percent error results do not exist for both ntCard and our model.

### 2.2 Percent Error Results with K-Mer Length 64

| ERR | Hash 1 (City) |  |  |  |  | AVG | STDEV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \% | $\checkmark$ | 8 | 10 | 12 | 14 |  |  |
| aplCal1 | 28,32903 | 9,738833 | 1,081687 | 0,17514 | -0,87483 | 4,360159 | 13,31973 |
| droEre2 | -12,6461 | -0,30178 | 0,922904 | -0,11187 | -1,04152 | -13,1783 | 5,044385 |
| triCas2 | 16,57566 | -2,22379 | 1,063134 | 0,069423 | 1,32826 | 3,362537 | 7,517709 |
| est | 57,97755 | 31,57177 | 23,95072 | 21,00053 | 21,82709 | 31,26553 | 15,50473 |
| est1 | 24,54762 | 8,810022 | 16,38134 | 14,90347 | 15,7816 | 16,08481 | 5,615094 |
| danRer |  |  |  |  |  |  |  |

Table 6: 64 K-Mer Length City Hash Results


Table 7: 64 K-Mer Length Spooky Hash Results

| Hash 3 (Murmur) |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\%$ | 6 | 8 | 10 | 12 | 14 | AVG | STDEV |
| aplCal1 | $-0,7054$ | 0,067993 | $-2,07026$ | $-0,90465$ | $-0,7054$ | $-0,86354$ | 0,770653 |
| droEre2 | 6,591349 | 5,772626 | $-5,32064$ | $-2,24536$ | $-2,02832$ | 0,553931 | 5,308008 |
| triCas2 | 44,81134 | 20,39261 | 4,395721 | 1,134941 | 1,038187 | 14,35456 | 18,81048 |
| est | 23,28877 | 16,93833 | 20,65375 | 23,33741 | 23,97426 | 21,6385 | 2,920906 |
| est1 | 6,996839 | 23,62446 | 15,80245 | 16,66608 | 15,67019 | 15,752 | 5,90494 |
| danRer |  |  |  |  |  |  |  |

Table 8: 64 K-Mer Length Murmur Hash Results

| Hash 4(Tabulation) |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\%$ | 6 | 8 | 10 | 12 | 14 | AVG | STDEV |
| apICal1 | $-1,30217$ | 9,532488 | $-1,5017$ | $-1,32744$ | $-1,70005$ | 0,740226 | 4,917613 |
| droEre2 | $-17,1448$ | $-12,1746$ | $-4,26843$ | $-0,41496$ | $-0,8145$ | $-6,96347$ | 7,395477 |
| triCas2 | 3,139113 | 6,0111 | 3,121772 | 1,547118 | 1,236432 | 3,011107 | 1,892176 |
| est | 29,68183 | 21,32712 | 24,4255 | 21,82873 | 26,39814 | 24,73226 | 3,442129 |
| est1 <br> danRer | 26,64757 | 4,340913 | 18,38749 | 14,22262 | 14,65305 | 15,65033 | 8,052439 |

Table 9: 64 K-Mer Length Tabulation Hash Results
As in the $\mathrm{K}=32$ results, est and est1 results are still not reliable and therefore it is hard to make an outcome. NtCard also gives high erroneous estimations about est and estl datasets. Except for Spooky Hash, all other hash functions give much more accurate results on $K=64$ than $K=32$. This indicates that the conclusion of greater K length leads more accurate results. There is no error higher than $\% 2$ when Tabulation Hash is performed and bucket bit number is 14 . When the bucket bit is 6 , poor results are likely to arise. Improvement of accuracy of Murmur can be seen in Table 8.

## 3 Conclusion and Future Work

Findings so far suggest that HyperLogLog data sketch for K-mer counting gives more accurate results with bucket bits around 10 and a large distinct element size utilizing Tabulation and City Hashes, yet there are other conditions to consider. Right now, even though our implementation of HyperLogLog saves a lot of space, it runs sequentially, therefore it takes time to process the datasets; parallelization is needed to bring our model to more desirable speeds. Also, in our estimation formula, there is a correction constant that differs for each bucket size which might need some tweaking. Although a file size of 1 GB , like the ones we used might seem large at first, file sizes can go much larger in the field of Bioinformatics. Tests need to be done on these types of larger data to have more accurate results and to see which hash function performs the best for HyperLogLog sketch.

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## Appendices

## Appendix A

Error Results with K-Mer Length 40

| $\begin{array}{\|l\|} \hline \text { ERR } \\ \hline \% \\ \hline \end{array}$ | Hash 1 (City) |  |  |  |  | AVG | STDEV | \% | Hash 2 (Spooky) |  |  |  |  | AVG | STDEV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6 - | 8 | 10 | 12 | 14 |  |  |  | 6 - | 8 - | 10 | 12 | 14 |  |  |
| aplCal1 | 28.329031 | 13.810239 | -3.20215 | -0.43761 | -1.33368 | 4.360159 | 13.31973 | aplCal1 | 7.407527 | 5.92531 | 2.9053829 | 1.013237 | -0.70755 | 3.308781 | 3.361886 |
| droEre2 | 9.83139 | 6.617879 | 5.297024 | 1.503681 | -1.16117 | 22.0888 | 3.86102 | droEre2 | -13.563 | -6.45241 | -3.082123 | -2.1849 | -0.88737 | -5.23396 | 5.091102 |
| triCas2 | -15.96679 | 3.33533 | 7.741705 | 4.51567 | 2.593122 | 0.443808 | 9.382529 | triCas2 | 5.761526 | -6.40233 | 0.1031843 | 3.010521 | 0.732663 | 0.641113 | 4.520525 |
| est | -6.201545 | 8.24261 | 18.21004 | 24.30403 | 22.08291 | 13.32761 | 12.53216 | est | 8.054934 | 17.53015 | 25.255193 | 26.90899 | 23.7332 | 20.29649 | 7.707679 |
| est1 | -14.3671 | 111.91766 | 10.22176 | 13.81715 | 16.20329 | 7.558552 | 12.45723 | est | 11.58477 | 7.487913 | 14.843526 | 13.4085 | 16.96096 | 12.85713 | 3.589518 |
| danRer | 4.8578846 | 12.58754 | 5.718625 | 3.807725 | 4.840399 | 6.362434 | 3.545179 | danRer | -10.7519 | -4.6079 | 6.3590817 | 6.194416 | 6.499414 | 0.738615 | 7.986868 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | sh 4 (Murmur) |  |  |  |  |  |  | Has | h 5 (Tabulation) |  |  |  |  |
| \% | 6 | 8 | 10 | 12 | 14 | AVG | StDEV | \% | 6 | 8 | 10 | 12 | 14 | AVG | STDEV |
| aplCal1 | 13.86492 | -7.66097 | -1.17478 | -3.30619 | -3.654601 | -0.38632 | 8.304327 | aplCal1 | 13.04263 | -16.6824 | -5.004373 | -3.45269 | -2.65846 | -2.95105 | 10.59363 |
| droEre2 | -12.6859 | -7.34543 | -6.49402 | -3.63491 | -2.790242 | -6.5901 | 3.902259 | droEre2 | 0.100119 | 5.314277 | -0.661269 | -2.23048 | -0.11461 | 0.481606 | 2.851198 |
| triCas2 | 8.846595 | -4.16029 | 3.705522 | 3.109671 | 1.2542494 | 2.551149 | 4.690485 | triCas2 | 19.47696 | 0.879472 | -1.003382 | 4.196165 | 2.511616 | 5.212167 | 8.203884 |
| est | 29.11144 | 31.49208 | 22.81277 | 22.7882 | 23.077415 | 25.85638 | 4.145975 | est | 41.38791 | 29.35921 | 19.959875 | 19.76241 | 22.1542 | 26.52472 | 9.177095 |
| est1 | 19.26702 | 7.413903 | 16.26557 | 13.61818 | 14.513049 | 14.21555 | 4.371148 | est1 | 19.26702 | 7.413903 | 19.175086 | 13.98109 | 13.49226 | 14.66587 | 4.897473 |
| danRer | 9.112314 | 10.31699 | 7.754392 | 4.966647 | 5.7694932 | 7.583967 | 2.23478 | danRer | 24.07031 | 1.914857 | 2.1477857 | 5.134403 | 4.502338 | 7.553939 | 9.34054 |

Error Results with K-Mer Length 48


Error Results with K-Mer Length 56


## Appendix B

Bucket Bit - Absolute Error Plots with K-Mer Length 32


Bucket Bit - Absolute Error Plots with K-Mer Length 40


Bucket Bit - Absolute Error Plots with K-Mer Length 48


Bucket Bit - Absolute Error Plots with K-Mer Length 56


Bucket Bit - Absolute Error Plots with K-Mer Length 64
(Mash 1 (City)

